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PRESSURE DROP INDUCED BY A SPHERE SETTLING IN NON-NEWTONIAN FLUIDS

G. S. RIBEIRO,† A. S. VARGAS and M. N. FROTA

Departamento de Engenharia Mecânica, Pontifícia Universidade Católica do Rio de Janeiro, 22453 Rio de Janeiro RJ, Brazil

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Abstract—This paper describes experiments related to the settling of a small solid particle in quiescent non-Newtonian fluids which are confined in a circular duct. Measurements of the pressure drop created by the settling particle were conducted, in order to verify the validity of a conjecture by Brenner. This conjecture, already confirmed for Newtonian fluids, predicts that for a very small particle flowing in the Stokes and Oseen regimes, the pressure drop force would not be equal to the drag force on the sphere, as would be suggested by momentum considerations in the limiting situation of an "unbounded" fluid. The present experiments indicate that the conjecture is valid for the case of non-viscoelastic power law fluids. For fluids exhibiting normal stresses effects and a power-law viscosity function, the validity of Brenner's results depends on the balance of normal and viscous stresses. The predictions seem to hold for situations where the effects of normal stresses are small compared to those of viscous stresses.

Key Words: particle sedimentation, non-Newtonian fluid

1. INTRODUCTION

Brenner (1962) showed that for low Reynolds numbers flows past a small particle, the forces that act over distanct duct walls are not negligible in comparison to the drag forces on the particle. In the case of a single particle falling in a Newtonian fluid in the Oseen regime, the forces on the wall are of the same order of magnitude as the drag on the particle, even for the case of "unbounded flows", with "infinitely distant" walls.

Brenner's calculations involve the rate of dissipation of mechanical energy. The particle is modeled as a perturbation to the flow. Its effects are computed by the difference between the viscous dissipation in the flow in the presence of the particle (*disturbed flow*) and that in a flow with the same average velocity but without the particle (*undisturbed flow*). Then, the "additional dissipation" due to the presence of the particle is determined. This additional dissipation rate may be obtained either from the governing equations of the disturbed fluid motion or directly from an energy balance. By comparing the solutions from both methods, Brenner's remarkable result allows the parameters of the *disturbed flow* to be evaluated by means of parameters of the *undisturbed flow*. As the total dissipation must be related to the pressure drop, Brenner obtained the following result:

$$\frac{\Delta P^{+} A}{D} = \frac{v^{0}}{V_{\mathrm{m}}}, \qquad [1]$$

where v^0 is the velocity of the undisturbed flow measured at the center of mass of the particle in the disturbed flow, V_{m} is the average velocity (assumed equal in the disturbed and undisturbed flow), A is the cross section area of the duct, D is the viscous drag on the particle and ΔP^{+} is the additional pressure drop due to the presence of the particle. It is interesting to observe that in [1] the right-hand side contains only parameters of the undisturbed flow, while the left-hand side displays the ratio between the additional pressure force (arising from the presence of the particle) and the viscous drag.

†Present address: Department of Aerospace Engineering and Mechanics, University of Minnesota, MN 55455, U.S.A.

It is easy to show now from a balance of forces that the additional force exerted over the walls by the fluid, F_W^+, is

$$F_W^+ = \left(\frac{v^0}{V_m} - 1\right)D. \qquad [2]$$

By a Galilean transformation, these results can be applied to a particle settling through a quiescent fluid, contained within a duct. Then, for the case of a particle settling along the axis of a circular tube, it is possible to calculate the values predicted by [1] and [2] from the velocity profile of a Poiseuille flow:

$$\frac{v^0}{V_m} = 2, \quad F_W^+ = D, \quad \frac{\Delta P^+ A}{D} = 2. \qquad [3]$$

These results were verified experimentally by Pliskin & Brenner (1963) and Feldman & Brenner (1968), who studied the relative motion between a sphere and a Newtonian fluid confined in a circular cylinder. Langins *et al.* (1971) considered conical particles and Pereira & Frota (1986) investigated the case of non-circular ducts. These works also indicated that Brenner's results were still valid at particle Reynolds numbers as high as 120, far beyond the range of Oseen's regime.

Since [1] and [2] do not include the physical properties of the fluid, Brenner conjectured that these equations could be used for non-Newtonian fluids as well, even though the Newtonian constitutive equation was used to compute the dissipation energy and to derive [1]. What follows is a simple calculation, proposed by Brenner (1962). For a power-law fluid, Fredrikson & Bird (1958) computed the velocity profile in axial flow between concentric cylinders. By a power-law fluid, it is meant a fluid in which the only stresses other than pressure are shear stresses of the form $\tau = m\dot{\gamma}^n$, where $\dot{\gamma}$ is the shear rate. The inner cylinder is considered a "particle" in the flow. In the power-law axial flow between concentric cylinders, the pressure drop and the drag on the inner cylinder per unit length are, respectively:

$$\frac{\Delta P}{l} = \frac{2m}{R_o}\left(\frac{s+3}{R_o}V_m\right)^{1/s}\left[1 + \left(\frac{s+3}{s+1}\right)\left(\frac{s-1}{s+1}\right)^{1/s}\left(\frac{R_i}{R_o}\right)^{(s-1)/s}\right] \qquad [4]$$

and

$$\frac{D}{l} = 2\pi m R_o\left(\frac{s+3}{R_o}V_m\right)^{1/s}\left[\left(\frac{s-1}{s+1}\right)^{1/s}\left(\frac{R_i}{R_o}\right)^{(s-1)/s}\right]. \qquad [5]$$

where R_o is the external radius, R_i is the internal radius and s is the inverse of the power-law exponent n ($s = 1/n$).

Now, the undisturbed velocity field and pressure drop in laminar flow of a power-law fluid in a circular duct are:

$$\frac{v}{V_m} = \left(\frac{s+3}{s+1}\right)\left[1 - \left(\frac{r}{R_o}\right)^{(s+1)}\right] \qquad [6]$$

and

$$\frac{\Delta P^0}{l} = \frac{2m}{R_o}\left(\frac{s+3}{R_o}V_m\right)^{1/s}. \qquad [7]$$

So, one can compute from [6] the undisturbed fluid velocity at the position of the "particle" (center of the duct) in the disturbed flow as

$$\frac{v^0}{V_m} = \frac{s+3}{s+1}. \qquad [8]$$

One can also obtain the extra pressure drop due to the presence of an inner cylinder ("particle") by subtracting [7] from [4]:

$$\frac{\Delta P^+}{l} = \frac{2m}{R_o}\left(\frac{s+3}{R_o}V_m\right)^{1/s}\left[\left(\frac{s+3}{s+1}\right)\left(\frac{s-1}{s+1}\right)^{1/s}\left(\frac{R_i}{R_o}\right)^{(s-1)/s}\right]. \qquad [9]$$

Combining [5] and [9] gives:

$$\frac{\Delta P^+ A}{D} = \frac{s+3}{s+1} = \frac{v^0}{V_m}, \qquad [10]$$

which is a statement of what is called here Brenner's conjecture for power-law fluids. From [8] and [10] it is possible to conclude that:

$$\frac{\Delta P^+ A}{D} = \frac{v^0}{V_m}; \quad F_W^+ = \left(\frac{v^0}{V_m} - 1\right)D. \qquad [11]$$

These are the same results as for the case of Newtonian fluids. From these simple calculations one could expect that, at least for power-law fluids, Brenner's conjecture would be valid.

The purpose of this paper is to verify experimentally the extension of Brenner's results for power-law fluids (generalized Newtonian fluids) and for more general classes of non-Newtonian fluids. The experiments consisted of dropping small spheres along the axis of a circular duct filled by an otherwise quiescent fluid and measuring the pressure drop created by the particle.

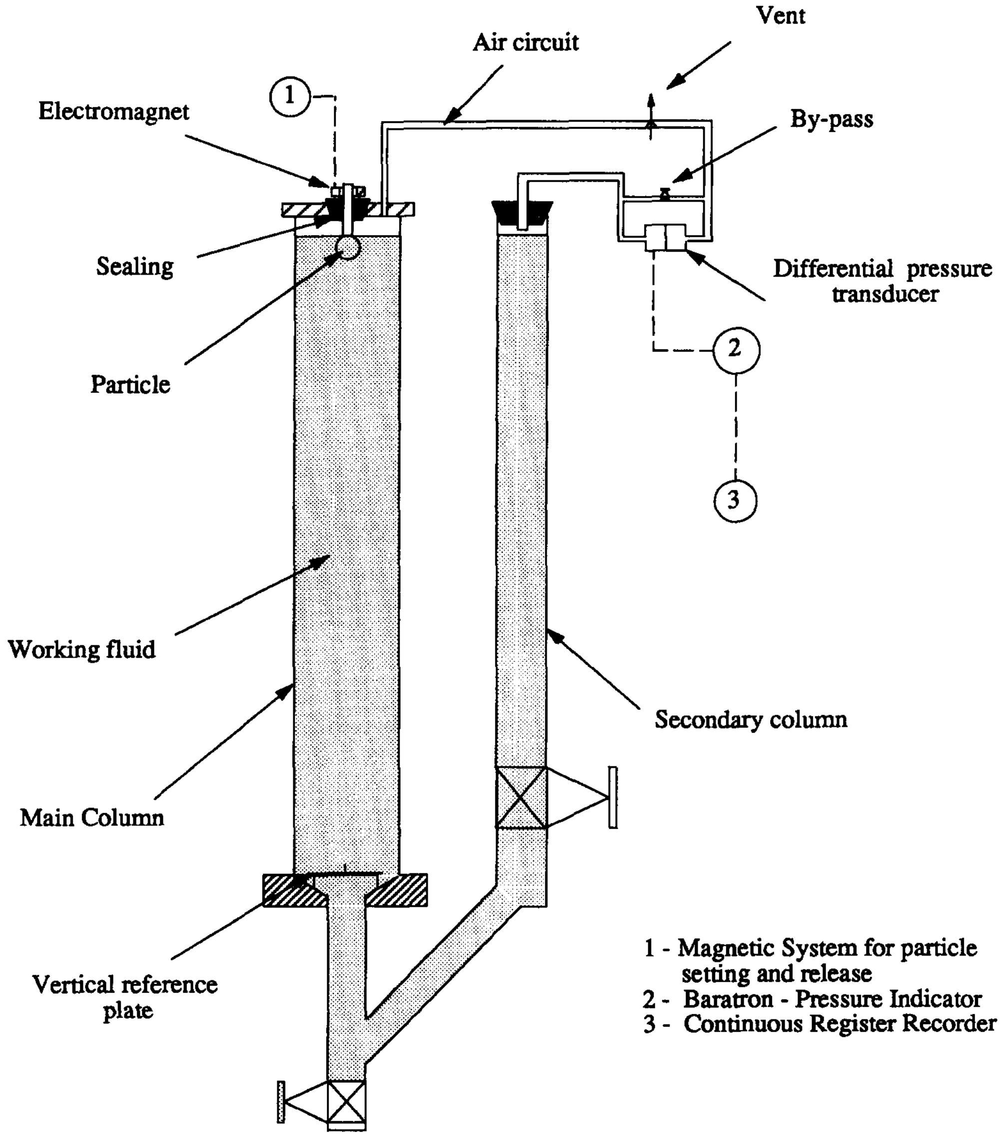

Figure 1. Experimental apparatus.

2. EXPERIMENTAL PROCEDURE

Figure 1 shows the experimental apparatus built for the evaluation of the ratio $(\Delta P^+ A)/D$. The system consists basically of two interconnected vertical columns filled with the working fluid. The sphere is released from the top of the main column, which is precisely vertical, by a magnetic device. As the particle accelerates, it causes the liquid level to fall in the main column and to rise in the other one. A differential pressure transducer, connected to the entrapped air volumes on top of both columns, directly measures the pressure drop induced by the settling particle. A continuous recorder gives a register of the transducer reading during experiment. The pressure measurements were taken after the particle reached its terminal velocity, when the transducer reading was a constant pressure expressed by a plateau in the strip-chart record. Fully developed flow typically occurred after 20–30 sphere diameters from the releasing point, depending on the fluid. Similar apparatus was used in previous experimental work for verifying Brenner's results (Pliskin & Brenner 1963; Feldman & Brenner 1968).

The drag force at steady state is calculated by a simple force balance as the apparent weight of the sphere. The particle Reynolds number is calculated by

$$\mathrm{Re} = \frac{\rho v^{2-n} d^n}{8^{n-1} m \left(\frac{3n+1}{4}\right)^n}, \qquad [12]$$

where ρ is the fluid density, m and n are the power-law viscosity parameters, d is the particle diameter and v is the terminal velocity of the particle.

The diameter of the main column was 76.9 mm, and the spheres fell through a distance of 1388 mm. The particles used were precision stainless-steel spheres. In most experiments, we used particle diameters ranging between 3.175–10.00 mm. In some cases, bigger spheres (12.70–19.05 mm) were used. The parameter a/R_0 is the ratio between the radii of the particle and the column, respectively, and ranged between 0.041 and 0.247. In Newtonian fluids, the wall effects on the ratio $(\Delta P^+ A)/D$ are expected to be low for small particles (up to 1% for $a/R_0 < 0.10$, between 1–4% for $0.10 < a/R_0 < 0.25$). We do not known for sure the importance of wall effects in the case of non-Newtonian fluids, to minimize them we tried to work with small spheres whenever possible.

To protect the column system from external perturbations, it was mounted on top of a massive structure inside a room with controlled temperature. In this way we managed to reduce considerably the effects of external vibrations but, as registered in previous work, a drift in the zero reading of the transducer during certain experiments was observed. This thermal drift, which is a consequence of very small temperature changes in the two air volumes that transmit the pressure to the transducer, showed a linear drift rate for the duration of an experiment, allowing us to determine the pressure reading in the same way as done before by Feldman & Brenner (1968) and others.

In order to qualify the experimental procedure, some experiments were done with a Newtonian fluid (glycerol, 98% in distilled water). In these tests, the theoretical values foreseen for the parameter $(\Delta P^+ A)/D$ were reproduced within the experimental uncertainty, estimated to be 2.5%.

3. EXPERIMENTAL RESULTS

Two different series of experiments were performed. The first one considered a non-viscoelastic power-law fluid (generalized Newtonian fluid). Later, viscoelastic power-law fluids were investigated.

3.1. Experiments with a non-viscoelastic power-law fluid

The working liquid used was a solution of polyacrylic acid (CARBOPOL 940): 3000 wppm—in glycerol 98%. This fluid is known not to have normal stress effects, and in tests did not climb in a rotating rod, confirming the fact above. The fluid density, measured at the temperature of the experiments (27.0 ± 0.5°C) was 1253 kg/m^3. Its power-law parameters, measured in a Ferranti–Shirley cone-and-plate viscometer at the same temperature were: $n = 0.874$ and $m = 1.30$ Pa s^n.

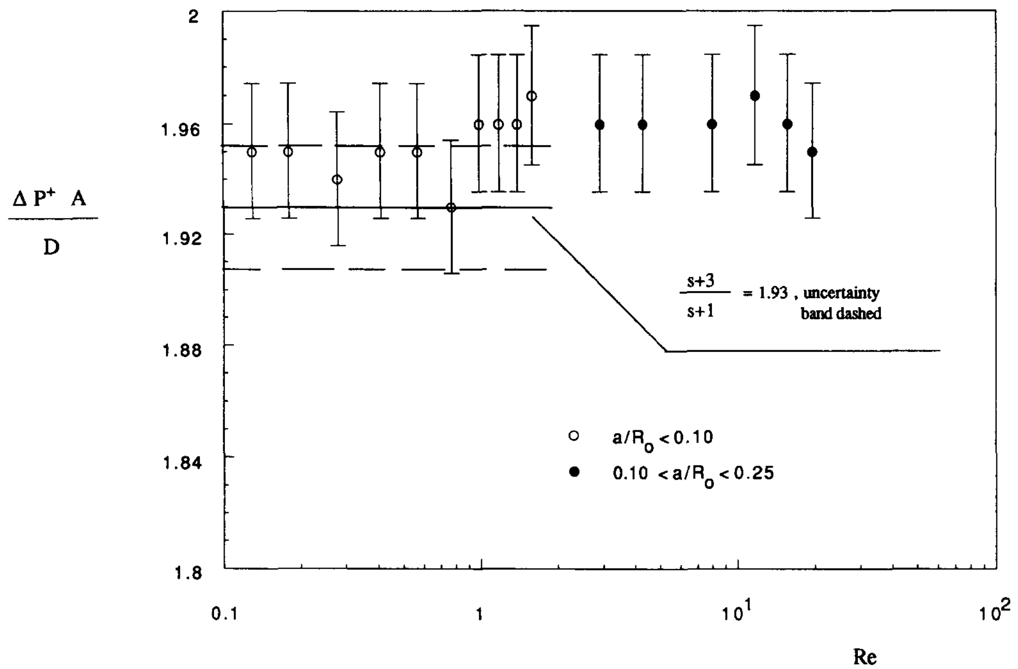

Figure 2. Experimental results for a non-viscoelastic power-law fluid.

The experimental results obtained for power-law fluids are collated in figure 2. The fluid used has an n value of 0.874, which gives a threoretical value of 1.93 for the parameter $(\Delta P^+A)/D$ computed from [10].

Observing figure 2, one can see the good agreement between the experimental results and the theoretical value proposed in Brenner's modified model. The solid line shows the reference theoretical value, 1.93, limited to low values of Re (<2), as it would be in Oseen's regime for Newtonian flows. For the experiments in this range, one can obtain the average value of 1.95 (with a 0.01 variance), 1% greater than the theoretical value of 1.93. It is important to emphasize that beyond the experimental uncertainties observed in $(\Delta P^+A)/D$ (in the order of 2.5%), there are also uncertainties associated with the experimental determination of the ratio $(s + 3)/(s + 1)$ (estimated at 1%, and displayed in figure 2 as a band between the dashed lines).

Other experiments were also made with $\mathrm{Re} > 2$, as shown in figure 2. It can be seen that, as in the Newtonian case, Brenner's results seem to be valid beyond Oseen's regime. The largest value of Re checked was approximately 20. In these experiments, the average value of the parameter $(\Delta P^+A)/D$ measured was 1.96 (with a variance of 0.01).

All experiments with the non-viscoelastic power-law fluid were reproducible within, at most, 1%.

3.2. *Experiments with viscoelastic power-law fluids*

Experiments with fluids showing normal stress effects were performed with distilled water solutions of polyacrylamide (SEPARAN AP-273). Table 1 presents the values of the physical properties of these fluids, at a temperature of $24 \pm 1^\circ\mathrm{C}$, with which the tests were performed.

The measurement of the viscosity function was made with the Ferranti–Shirley cone-and-plate viscometer and fitted to a power-law form. The fluid characteristic time, λ, was estimated following

Table 1. Physical properties of aqueous solutions of polyacrylamide (SEPARAN AP-273)

Concentration (wppm)	ρ (kg/m^3)	n	m (Pa s^n)	λ(s)
10,000	998	0.303	8.1	133.2
5000	997	0.343	2.7	53.3
3000	998	0.483	1.1	25.6

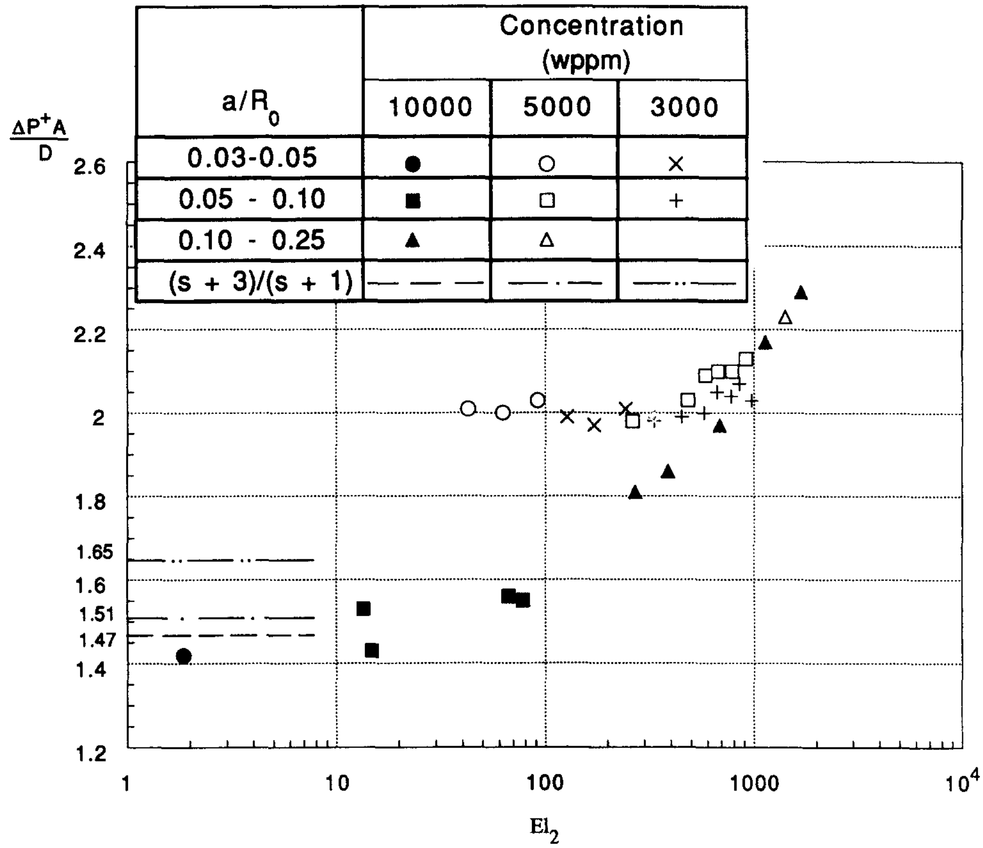

Figure 3. Experimental results for viscoelastic power-law fluids.

the Powell–Eyring model. All these fluids showed a very distinct climb when a rotating rod was immersed in them.

The experimental results are presented in figure 3. Each point in the graph corresponds to the average value of 5 repetitions of the same experiment. The graph shows the variation of the hydrodynamic parameter $(\Delta P^{+}A)/D$ with the dimensionless parameter El_2, the so-called second elastic number according to Astarita & Marucci (1974), for different ranges of the aspect ratio, a/R_0, the ratio of the sphere radius to the duct radius.

El_2 is calculated as the geometric average of Re and the Weissenberg number, $We = \lambda\dot{\gamma}$:

$$El_2 = \sqrt{\mathrm{Re\,We}}. \qquad [13]$$

El_2 reflects a ratio between normal and initial forces over viscous forces. High values of El_2 and low Re mean a predominance of the normal stress effects over the viscous effects. Correspondingly, low values of El_2 indicate the predominance of the viscous (tangential stress) effects.

Observing figure 3, one can see that, for the lowest values of El_2, the experimental values of the hydrodynamic parameter $(\Delta P^{+}A)/D$ apparently fit the value $(s + 3)(s + 1)$, which was suggested for a power-law fluid. As the values of El_2 increase, the parameter $(\Delta P^{+}A)/D$ assumes values around 2. In these experiments, the Re varied between 0.1 and 90. The value of We varied between 850 and 38,000.

The experiments at low El_2 were performed for very small settling velocities and they took a long time (up to 10 min). Because of thermal drift and other external effects, it was much harder to measure the pressure read by the transducer. These experiments had a reproducibility no greater than 4.5%, and there is a pronounced scatter among these points. Experiments at higher values of El_2 could be reproduced within 2%.

4. CONCLUSION

The results displayed in figure 2 seem to confirm Brenner's conjecture about the extension of his model to non-viscoelastic power-law fluids, in spite of the strong restriction imposed by the linearity of the stress tensor used in the calculation of the dissipation energy. As was noticed by other researchers for Newtonian fluids, apparently it is also possible to extend the validity of Brenner's conjecture for Re outside the Oseen regime.

The experiments also lead to the conclusion that the extension of Brenner's conjecture for viscoelastic power-law fluids depends on the balance between inertial, normal and viscous forces. When the normal stress effects are negligible in the face of viscous stress effects, it seems that the extension of Brenner's model is possible. Otherwise, with larger normal stress effects, the experimental values found for the parameter $(\Delta P^{+}A)/D$ do not agree with Brenner's predictions for power-law fluids.

Another way of interpreting these results is in terms of shear wave propagation phenomenon. The parameter El_2 can also be viewed as a Mach number for the viscoelastic flow, representing the ratio between flow velocity and the shear wave speed characteristic of the fluid (Astarita & Marucci 1974). The Mach number determines the type of the vorticity equation, and a change of type in this equation may imply an alteration to the physics of the flow. Many authors report different experiments where modifications in the flow structure are observed under such circumstances (Joseph 1992). For instance, Liu & Joseph (1993) describe a change in the axis of orientation of sedimenting cylindrical particles in viscoelastic polymeric solutions which appear to occur at Mach numbers close to 1.

Under this point of view, a possible line of reasoning would be that Brenner's predictions are valid as long as the flow is elliptic. As the flow velocity increases past the shear wave speed, it becomes hyperbolic and Brenner's results do not hold anymore. The results displayed in figure 3 may suggest this fact, if it is taken in consideration that the characteristic time for the viscoelastic fluids (from which the shear wave speed may be calculated, for a viscoelastic Maxwell fluid) was estimated and not actually measured. This could explain the transition happening for values of $El_2 \gg 1$. Nevertheless, although the present results can suggest this hypothesis, they are not sufficient to establish it.

These results so far indicate that as long as the non-linearity in the fluid constitutive equations is only in the dependence of the viscosity with shear rate, Brenner's conjecture is right. With more complex forms of non-linearity it may no longer be valid, even for fluids with a viscosity function of the power-law type but presenting normal stress effects. In any case, further investigation is needed in order to establish a complete understanding of the problem.

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